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Modeling of an Interval Type-2 Neutrosophic Bézier Surface by Using Interpolation Method

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Abstract

A generalization of the type-1 neutrosophic set, interval type-2 fuzzy set, and intuitionistic fuzzy set is the interval type-2 neutrosophic set (IT2NS). This study will show how to use the interpolation approach to visualize the interval type-2 neutrosophic Bézier surface (IT2NBS) model. However, the existence of truth, indeterminacy, and falsity membership functions in neutrosophic features makes the model challenging to visualize. Apart from that, the attributes of IT2NS have an upper and lower bound, which makes it difficult. Using the IT2NS theory, this study will first introduce an interval type-2 neutrosophic control net relation (IT2NCNR) to build the model. The IT2NBS models will be represented by blending the IT2NCNR and the Bernstein basis function. Afterward, the truth, indeterminacy, and falsity memberships of the IT2NCNR are interpolated for both the upper and lower bounds to show the IT2NBSs. A review of the algorithm used to create the IT2NBS interpolation models will wrap up the study. Fortunately, the results of this study will yield a predictive model that is used in many medical applications, such as picture blurring detection, and cancer level prediction.

Keywords: *Interval Type-2 Neutrosophic Set, Control Net, Bézier Surface, Interpolation Model and Upper and Lower Bound.*

1. Introduction

Zadeh (1965) developed fuzzy set (FS) theory as a generalization of the classical concept of a set and a proposition to account for fuzziness (degree of truth) as expressed in natural or human language (Pranevicius et al., 2014; Chiu, 1997). Although type-1 FS (T1FS) is commonly used and has an associated meaning of uncertainty, previous research has demonstrated that T1FS only partially depicts uncertainty and may be unable to handle or decrease the effects of uncertainties found in some real-world applications (Mendel, 2001). To address this issue, Zadeh (1975) proposed expanding his earlier T1FS theory to include the type-2 fuzzy set (T2FS) theory, which can handle uncertainties that T1 finds difficult to manage due to T2FS's fuzzy membership grades. A few research have demonstrated that the outcome of a T2FS may be better than that of a T1FS (Hagras, 2004; Liang & Mendel, 2000; Melgarejo et al., 2004; Melin and Castillo, 2004; Mendel, 2001; Ozen and Garibaldi, 2003; Wu and Tan, 2004). Due to the computational cost of using a generic T2FS, most individuals now exclusively use interval T2FSs. The computations for interval T2FSs are manageable and practicable (Mendel, 2001).

Atanassov (1986) presented intuitionistic fuzzy sets (IFSs), accounting for membership and non-membership grades. Smarandache (2019) expanded IFSs to neutrosophic sets (NSs) with truth, indeterminacy, and falsity membership functions to capture the indeterminacy membership grade. Wang et al. (2010) suggested the concept of a single-valued neutrosophic set (SVNS) as a solution to ambiguous, indeterminate, and incoherent data. Wang et al. (2004) created interval type-2 neutrosophic sets (IT2NSs) to quantify improbability and ambiguity better (Ye, 2014). Touqeer et al. (2021) proposed interval type-2 trapezoidal neutrosophic numbers via operations. According to the study, an interval neutrosophic set is defined and used the same way as an interval type-2 neutrosophic set.

The data set is an important component for presenting the surfaces. If there is any ambiguity in a data collection, it must be resolved before being used to create surface models. Using an IT2NS to create geometric models is an effective technique to address the issue of making data obvious when there is uncertainty. Research has been conducted to ensure that curves and surfaces in geometric modeling with uncertainty data are easily usable (Yamaguchi 1988; Rogers 2001; Farin 2002; Piegl and Tiller 1995). There have been various academic works on fuzzy geometry modeling (Jacas et al., 1997; Wahab et al., 2004; Wahab et al., 2009; Wahab et al., 2010; Jacas et al., 1997; Bidin et al., 2022; Zakaria et al., 2021). Rosli and Zulkifly (2023a; 2023b; 2023c; 2023d; 2024a, 2024b, 2024c, 2024d, 2024e) recently proposed type-1 neutrosophic geometric modeling for B-spline curve interpolation, B-spline surface interpolation, 3-dimensional quartic Bézier curve approximation model, bicubic Bezier surface approximation, 3-dimensional B-spline surface approximation model, and interval neutrosophic cubic Bézier curve approximation model. Meanwhile, this paper will offer an interval type-2 neutrosophic Bézier surface using the interpolation approach.

The main objective of this research is to create a modeling of an interval type-2 neutrosophic Bézier surface (IT2NBS) interpolation model. Before building the IT2NBS, the IT2NS, and its properties must be used to define the interval type-2 neutrosophic control net relation (IT2NCNR). The Bernstein basis function is used with these control nets to create models of IT2NBSs, which are then visualized using an interpolation method. The following is how this paper is organized. Section 1 provided some background information regarding the study. Section 2 explains the fundamental concepts of IT2NS, interval type-2 data points (IT2DPs), and IT2NCNR. Section 3 describes how to use IT2NCNR to interpolate the IT2NBS. Section 4 includes a numerical example and a visualization of IT2NBS. The surface properties are also shown, along with the method by which they were created. Finally, part 5 will bring this study to a conclusion.

2. Basic Properties

This part aims to introduce the IT2NDPs and IT2NCPs that refer to a dataset, with the IT2NDPs being regarded as IT2NCPs to represent the IT2NBCs. Thus, before discussing IT2NDPs, it is necessary to define IT2NS, interval type-2 neutrosophic relation (IT2NS), and interval type-2 neutrosophic point (IT2NP). Wang et al. (2005) introduced the essential notion of IT2NS. Later, Tas and Topal (2017; 2018) inspired the definition of IT2NCPs for neutrosophic points. Furthermore, the concept of IT2NDPs comes from a study by Zakaria et al. (2013a; 2013b; 2013c) on type-2 fuzzy data points in geometric modeling and an interval type-2 trapezoidal neutrosophic numbers by (Touqeer et al., 2021).

Definition 1 (Wang et al., 2005)
Let X be the universal set that elements in X denoted as x. An interval type-2 neutrosophic set (IT2NS) A is expressed by the truth membership function T_A, indeterminacy membership function I_A, and false membership function F_A. Where $x \in X, T_A(x), I_A(x), F_A(x) \subseteq [0,1]$.

- When X is continuous, an IT2NS A can be expressed as

$$A = \int_X \langle T(x), I(x), F(x) \rangle / x, x \in X \tag{1}$$

- When X is discrete, an IT2NS A can be expressed as

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i), F(x_i) \rangle / x_i, x_i \in X \tag{2}$$

Definition 2 (Wang et al., 2005)
Suppose X and Y be a non-empty crisp set. $R(X,Y)$ denoted as interval type-2 neutrosophic relation (IT2NR) in a subset of product space $X \times Y$ and containing the truth membership function $T_R(x,y)$, indeterminacy membership function $I_R(x,y)$ and false membership function $F_R(x,y)$ where $x \in X$ and $y \in Y$, and $T_R(x,y), I_R(x,y), F_R(x,y) \subseteq [0,1]$.

Definition 3 (Rosli & Zulkifly, 2024b)

Suppose A in the space of $x \in X$ is an interval type-2 neutrosophic point (IT2NP) and $x = \{x_i\}$ is a set of IT2NPs where there exists $T_A(x) = [\sup(T_A), \inf(T_A)]: X \to [0,1]$ defining as the supremum and infimum of truth membership, $I_A(x) = [\sup(I_A), \inf(I_A)]: X \to [0,1]$ defining as the supremum and infimum of indeterminacy membership and $F_A(x) = [\sup(F_A), \inf(F_A)]: X \to [0,1]$ defining as the supremum and infimum of false membership where

$$T_A(x) = \begin{cases} 0 & \text{if } x_i \notin X \\ a \in (0,1) & \text{if } x_i \hat{\in} X \\ 1 & \text{if } x_i \in X \end{cases} \tag{3}$$

$$I_A(x) = \begin{cases} 0 & \text{if } x_i \notin X \\ b \in (0,1) & \text{if } x_i \hat{\in} X \\ 1 & \text{if } x_i \in X \end{cases} \tag{4}$$

$$F_A(x) = \begin{cases} 0 & \text{if } x_i \notin X \\ c \in (0,1) & \text{if } x_i \hat{\in} X \\ 1 & \text{if } x_i \in X \end{cases} \tag{5}$$

Definition 4 (Rosli & Zulkifly, 2024b)
Let $A = \{x | x \text{ interval type-2 neutrosophic point}\}$ and $D = \{D_i | D_i \text{ data point}\}$ is a set of interval type-2 neutrosophic data points with $D_i \in D \subset X$, where X is a universal set and $T_A(D_i) = [\sup(T_A), \inf(T_A)]: D \to [0,1]$ for truth membership function which defined as $T_A(D_i) = 1$, $I_A(D_i) = [\sup(I_A), \inf(I_A)]: D \to [0,1]$ for indeterminacy membership function defined as $I_A(D_i) = 1$, $F_A(D_i) = [\sup(F_A), \inf(F_A)]: D \to [0,1]$ for falsity membership function defined as $F_A(D_i) = 1$ and formulated by $D = \{(D_i, T_A(D_i), I_A(D_i) F_A(D_i) | D_i \in \square)\}$. Thus,

$$T_A(D_i) = \begin{cases} 0 & \text{if } D_i \notin X \\ a \in (0,1) & \text{if } D_i \hat{\in} X \\ 1 & \text{if } D_i \in X \end{cases} \tag{6}$$

$$I_A(D_i) = \begin{cases} 0 & \text{if } D_i \notin X \\ b \in (0,1) & \text{if } D_i \hat{\in} X \\ 1 & \text{if } D_i \in X \end{cases} \tag{7}$$

$$F_A(D_i) = \begin{cases} 0 & \text{if } D_i \notin X \\ c \in (0,1) & \text{if } D_i \hat{\in} X \\ 1 & \text{if } D_i \in X \end{cases} \tag{8}$$

For all i and the three memberships, $D_i = \langle D_i^L, D_i, D_i^R \rangle$ with $D_i^L = \langle D_i^{LL}, D_i^L, D_i^{LR} \rangle$ where D_i^{LL}, D_i^L and D_i^{LR} are left-left, left, and left-right of IT2NDP and $D_i^R = \langle D_i^{RL}, D_i^R, D_i^{RR} \rangle$ where D_i^{RL}, D_i^R, and D_i^{RR} are right-left, right, and right-right of IT2NDP respectively. **Figure 1** illustrates this with the green triangle representing truth, the blue triangle representing indeterminacy, and the red triangle representing falsehood memberships. The dashed triangle for each membership at the values $[D^L, D, D^R]$ represented the type-1 neutrosophic set for each membership. The upper and lower boundaries for each membership are given by values $\left[D^{LL},D,D^{RR}\right]$ and $\left[D^{LR},D,D^{RL}\right]$, respectively.

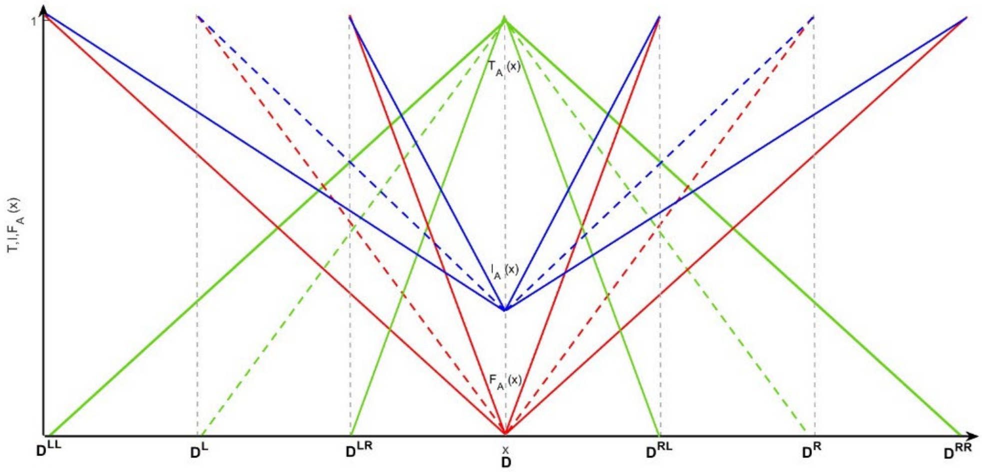

Figure 1 Interval type-2 Neutrosophic Data Points for Truth, indeterminacy, and Membership Degrees

2.1. Interval Type-2 Neutrosophic Control Net Relation (IT2NCPR)

The geometry of a spline surface can only be described by all the points required to build the surface. This is what the word "control net" means. The control net plays an important role in the development, control, and manufacture of smooth surfaces. The interval type-2 neutrosophic control point relation (IT2NCPR) is defined in this section by first using the notion of an interval type-2 neutrosophic set from the research published by (Wahab et al (2009); Wahab et al (2010)) in the following way:

Definition 5 (Rosli & Zulkifly, 2024b) Let $\hat{R}$ be an IT2NPR, then IT2NCPR is defined as a set of point $n+1$ that indicates the positions and coordinates of a location is used to describe the curve and is denoted by

$$\begin{aligned}\hat{P}_i^T &= \left\{\hat{p}_0^T,\hat{p}_1^T,\ldots,\hat{p}_n^T\right\}\\ \hat{P}_i^I &= \left\{\hat{p}_0^I,\hat{p}_1^I,\ldots,\hat{p}_n^I\right\}\\ \hat{P}_i^F &= \left\{\hat{p}_0^F,\hat{p}_1^F,\ldots,\hat{p}_n^F\right\}\end{aligned} \tag{9}$$

where $\hat{P}_i^T$, $\hat{P}_i^I$ and $\hat{P}_i^F$ are interval type-2 neutrosophic control points for membership truth, indeterminacy and i is one less than n. Thus, the IT2NCNR can be defined as follows.

Definition 6 Let $\hat{P}$ be an IT2NCPR, and then define an IT2NCNR as points n and m for $\hat{P}$ in their direction, and it can be denoted by $\hat{P}_{i,j}$ that represents the locations of points used to describe the surface and may be written as

$$\hat{P}_{i,j}^{T,I,F} = \begin{bmatrix}\hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,m}\\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,m}\\ \vdots & \vdots & \ddots & \vdots\\ \hat{P}_{n,0} & \hat{P}_{n,1} & \cdots & \hat{P}_{n,m}\end{bmatrix} \tag{10}$$

3. Interpolation of an Interval Type-2 Neutrosophic Bézier Surface (IT2NBS)

Surface is a vector value function with two parameters that govern how the plane is projected into the Euclidean three-dimensional frame (Piegl and Tiller, 1995). This type of mapping is known as surface mapping. The tensor product technique is a bidirectional curve construction method that employs basic functions as well as geometric coefficients. The IT2NCNR and Definition 1 are used to build the IT2NBS, which is then used to incorporate the Bézier blending function into a geometric model. Following that, it investigates the characteristics of the IT2NBS model. IT2NBS, which stands for interpolation approach, can be represented mathematically as follows:

Definition 7

Let $$\hat{P}^{T,I,F}_{i,j} = \begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,m} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{n,0} & \hat{P}_{n,1} & \cdots & \hat{P}_{n,m} \end{bmatrix}$$

where $i = 0,1,...,n$ and $j = 0,1,...,m$ is IT2NCNR. Cartesian Bézier surface is given by:

$$\begin{aligned} BS^T(u,w) &= \sum_{i=0}^{n}\sum_{j=0}^{m} \hat{P}^T_{i,j} C_i^n(u) C_j^m(w) \\ BS^I(u,w) &= \sum_{i=0}^{n}\sum_{j=0}^{m} \hat{P}^I_{i,j} C_i^n(u) C_j^m(w) \\ BS^F(u,w) &= \sum_{i=0}^{n}\sum_{j=0}^{m} \hat{P}^F_{i,j} C_i^n(u) C_j^m(w) \end{aligned} \tag{11}$$

where $C_i^n(u)$ and $C_j^m(w)$ are the Bernstein functions in the u and w parametric directions.

$$\begin{aligned} C_i^n(u) &= \binom{n}{i} u^i (1-u)^{n-i} \qquad (0)^0 \equiv 1 \\ C_j^m(w) &= \binom{m}{j} w^j (1-w)^{m-j} \qquad (0)^0 \equiv 1 \\ &\text{with} \\ \binom{n}{i} &= \frac{n!}{i!(n-1)!} \qquad (0)^0 \equiv 1 \\ \binom{m}{j} &= \frac{n!}{j!(m-1)!} \qquad (0)^0 \equiv 1 \end{aligned} \tag{12}$$

The surface for the IT2NBS will be in the IT2NCPR. As a result, the interpolation procedure is as follows:

$$\begin{bmatrix} \hat{F}_{0,0} & \hat{F}_{0,1} & \cdots & \hat{F}_{0,m} \\ \hat{F}_{1,0} & \hat{F}_{1,1} & \cdots & \hat{F}_{1,m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{F}_{n,0} & \hat{F}_{n,1} & \cdots & \hat{F}_{n,m} \end{bmatrix} = \begin{bmatrix} BS(u_0,w_0) & BS(u_0,w_1) & \cdots & BS(u_0,w_m) \\ BS(u_1,w_0) & BS(u_1,w_1) & \cdots & BS(u_1,w_m) \\ \vdots & \vdots & \ddots & \vdots \\ BS(u_n,w_0) & BS(u_n,w_1) & \cdots & BS(u_n,w_m) \end{bmatrix} \tag{13}$$

Each $BS(u_i, w_j)$ can be expressed as a matrix product as follows:

$$BS(u_i,w_j) = \begin{bmatrix} C_0^n(u_i) & C_1^n(u_i) & \cdots & C_n^n(u_i) \end{bmatrix} \times \begin{bmatrix} \hat{P}_{0,0} & \hat{P}_{0,1} & \cdots & \hat{P}_{0,m} \\ \hat{P}_{1,0} & \hat{P}_{1,1} & \cdots & \hat{P}_{1,m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{P}_{n,0} & \hat{P}_{n,1} & \cdots & \hat{P}_{n,m} \end{bmatrix} \times \begin{bmatrix} C_0^m(w_j) \\ C_1^m(w_j) \\ \vdots \\ C_m^m(w_j) \end{bmatrix} \tag{14}$$

All the separate equations can be combined into a single matrix equation:

$$\hat{F} = M^T \hat{P} N \tag{15}$$

where $\hat{F}$ denotes the given matrix data points from (13), and $\hat{P}$ denotes the matrix containing the unknown control points $\hat{P}_{i,j}$. The values of the Bernstein polynomials at the given parameters are contained in the matrix M^T and N:

$$M^T = \begin{bmatrix} C_0^n(u_0) & C_1^n(u_0) & \cdots & C_n^n(u_0) \\ C_0^n(u_1) & C_1^n(u_1) & \cdots & C_n^n(u_1) \\ \vdots & \vdots & \ddots & \vdots \\ C_0^n(u_i) & C_1^n(u_i) & \cdots & C_n^n(u_n) \end{bmatrix} \tag{16}$$

$$N = \begin{bmatrix} C_0^m(w_0) & C_0^m(w_0) & \cdots & C_0^m(w_m) \\ C_0^m(w_1) & C_1^m(w_1) & \cdots & C_1^m(w_m) \\ \vdots & \vdots & \ddots & \vdots \\ C_m^m(w_i) & C_m^m(w_i) & \cdots & C_m^m(w_m) \end{bmatrix} \tag{17}$$

Equation (15) can be easily decomposed into a series of linear systems. $P = M^T \hat{P}$ is defined first, then reduced to $\hat{F} = CN$ and simplified to

$$\hat{P} = \left(M^T\right)^{-1} \hat{F} \left(N\right)^{-1} \tag{18}$$

3.1. Properties of Interval Type-2 Neutrosophic Bézier Surface (IT2NBS) Interpolation

The surface-blending functions employ the Bernstein basis, therefore the Bézier surface has the same features. The primary characteristics of IT2NBS are as follows:

i. The degree of IT2NBS is always one less than the control net vertices in the direction that it is being measured in, regardless of the parametric direction.

ii. Control net vertices in a particular direction are two fewer than the continuity of the IT2NBS in that direction.

iii. The IT2NBS will conform to the IT2NCNR of the shape.
iv. The only points that coincide between the IT2NCNR and the IT2NBS that it generates are the corner points.
v. The IT2NBS is protected from outside interference by the IT2NCNR's convex hull.
vi. The IT2NBS does not show any signs of the quality known as variation declining. Undefined and unknown information exists on the variation-diminishing feature of bivariant IT2NBS.
vii. The IT2NBS does not change in any way when subjected to an affine transformation.

4. Visualization of Interval Type-2 Neutrosophic Bézier Surface (IT2NBS)

This section illustrates the IT2NBS interpolation model for the truth, indeterminacy, and falsity membership functions. This section will conclude with a demonstration of the combining of all memberships in one axis and a discussion of an algorithm for creating the IT2NBS. Let's examine **the matrixes below** as a dataset representing IT2NCN. Based on **Definition 4** and **Figure 1**, $\hat{P}_{3,3}$ as the mean IT2NCN, $\hat{P}^{L}_{3,3}$ as the left IT2NCN, $\hat{P}^{LL}_{3,3}$ as the left-left IT2NCN, $\hat{P}^{R}_{3,3}$ as the right IT2NCN and $\hat{P}^{RR}_{3,3}$ as the right-right IT2NCN for Bézier surfaces 4×4 IT2NCN and the degree of polynomial $n=3$.

$$\hat{P}_{3,3}=\begin{bmatrix} \langle(-17,17);0.4,0.7,0.2\rangle & \langle(-17,7);0.9,0.3,0.1\rangle & \langle(-17,-7);0.4,0.4,0.5\rangle & \langle(-17,-17);0.6,0.5,0.2\rangle \\ \langle(-7,17);0.6,0.4,0.3\rangle & \langle(-7,7);0.8,0.2,0.3\rangle & \langle(-7,-7);0.5,0.5,0.3\rangle & \langle(-7,-17);0.7,0.4,0.2\rangle \\ \langle(7,17);0.6,0.2,0.5\rangle & \langle(7,7);0.8,0.4,0.1\rangle & \langle(7,-7);0.5,0.7,0.1\rangle & \langle(7,-17);0.5,0.3,0.5\rangle \\ \langle(17,17);0.7,0.3,0.3\rangle & \langle(17,7);0.4,0.6,0.3\rangle & \langle(17,-7);0.4,0.6,0.3\rangle & \langle(17,-17);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{L}_{3,3}=\begin{bmatrix} \langle(-19,15)5;0.4,0.7,0.2\rangle & \langle(-19,5);0.9,0.3,0.1\rangle & \langle(-19,-9);0.4,0.4,0.5\rangle & \langle(-19,-19);0.6,0.5,0.2\rangle \\ \langle(-9,15);0.6,0.4,0.3\rangle & \langle(-9,5);0.8,0.2,0.3\rangle & \langle(-9,-9);0.5,0.5,0.3\rangle & \langle(-9,-19);0.7,0.4,0.2\rangle \\ \langle(5,15);0.6,0.2,0.5\rangle & \langle(5,5);0.8,0.4,0.1\rangle & \langle(5,-9);0.5,0.7,0.1\rangle & \langle(5,-19);0.5,0.3,0.5\rangle \\ \langle(15,15);0.7,0.3,0.3\rangle & \langle(15,5);0.4,0.6,0.3\rangle & \langle(15,-9);0.4,0.6,0.3\rangle & \langle(15,-19);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{LR}_{3,3}=\begin{bmatrix} \langle(-21,13)5;0.4,0.7,0.2\rangle & \langle(-21,3);0.9,0.3,0.1\rangle & \langle(-21,-11);0.4,0.4,0.5\rangle & \langle(-21,-21);0.6,0.5,0.2\rangle \\ \langle(-11,13);0.6,0.4,0.3\rangle & \langle(-11,3);0.8,0.2,0.3\rangle & \langle(-11,-11);0.5,0.5,0.3\rangle & \langle(-11,-21);0.7,0.4,0.2\rangle \\ \langle(3,13);0.6,0.2,0.5\rangle & \langle(3,3);0.8,0.4,0.1\rangle & \langle(3,-11);0.5,0.7,0.1\rangle & \langle(3,-21);0.5,0.3,0.5\rangle \\ \langle(13,13);0.7,0.3,0.3\rangle & \langle(13,3);0.4,0.6,0.3\rangle & \langle(13,-11);0.4,0.6,0.3\rangle & \langle(13,-21);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{LL}_{3,3}=\begin{bmatrix} \langle(-23,11)5;0.4,0.7,0.2\rangle & \langle(-23,1);0.9,0.3,0.1\rangle & \langle(-23,-13);0.4,0.4,0.5\rangle & \langle(-23,-23);0.6,0.5,0.2\rangle \\ \langle(-13,11);0.6,0.4,0.3\rangle & \langle(-13,1);0.8,0.2,0.3\rangle & \langle(-13,-13);0.5,0.5,0.3\rangle & \langle(-13,-23);0.7,0.4,0.2\rangle \\ \langle(1,11);0.6,0.2,0.5\rangle & \langle(1,1);0.8,0.4,0.1\rangle & \langle(1,-13);0.5,0.7,0.1\rangle & \langle(1,-23);0.5,0.3,0.5\rangle \\ \langle(11,11);0.7,0.3,0.3\rangle & \langle(11,1);0.4,0.6,0.3\rangle & \langle(11,-13);0.4,0.6,0.3\rangle & \langle(11,-23);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{R}_{3,3}=\begin{bmatrix} \langle(-15,19)5;0.4,0.7,0.2\rangle & \langle(-15,9);0.9,0.3,0.1\rangle & \langle(-15,-5);0.4,0.4,0.5\rangle & \langle(-15,-15);0.6,0.5,0.2\rangle \\ \langle(-5,19);0.6,0.4,0.3\rangle & \langle(-5,9);0.8,0.2,0.3\rangle & \langle(-5,-5);0.5,0.5,0.3\rangle & \langle(-5,-15);0.7,0.4,0.2\rangle \\ \langle(9,19);0.6,0.2,0.5\rangle & \langle(9,9);0.8,0.4,0.1\rangle & \langle(9,-5);0.5,0.7,0.1\rangle & \langle(9,-15);0.5,0.3,0.5\rangle \\ \langle(19,19);0.7,0.3,0.3\rangle & \langle(19,9);0.4,0.6,0.3\rangle & \langle(19,-5);0.4,0.6,0.3\rangle & \langle(19,-15);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{RL}_{3,3} = \begin{bmatrix} \langle(-13,21)5;0.4,0.7,0.2\rangle & \langle(-13,11);0.9,0.3,0.1\rangle & \langle(-13,-3);0.4,0.4,0.5\rangle & \langle(-13,-13);0.6,0.5,0.2\rangle \\ \langle(-3,21);0.6,0.4,0.3\rangle & \langle(-3,11);0.8,0.2,0.3\rangle & \langle(-3,-3);0.5,0.5,0.3\rangle & \langle(-3,-13);0.7,0.4,0.2\rangle \\ \langle(11,21);0.6,0.2,0.5\rangle & \langle(11,11);0.8,0.4,0.1\rangle & \langle(11,-3);0.5,0.7,0.1\rangle & \langle(11,-13);0.5,0.3,0.5\rangle \\ \langle(21,21);0.7,0.3,0.3\rangle & \langle(21,11);0.4,0.6,0.3\rangle & \langle(21,-3);0.4,0.6,0.3\rangle & \langle(21,-13);0.7,0.4,0.2\rangle \end{bmatrix}$$

$$\hat{P}^{RR}_{3,3} = \begin{bmatrix} \langle(-11,23)5;0.4,0.7,0.2\rangle & \langle(-11,13);0.9,0.3,0.1\rangle & \langle(-11,-1);0.4,0.4,0.5\rangle & \langle(-11,-11);0.6,0.5,0.2\rangle \\ \langle(-1,23);0.6,0.4,0.3\rangle & \langle(-1,13);0.8,0.2,0.3\rangle & \langle(-1,-1);0.5,0.5,0.3\rangle & \langle(-1,-11);0.7,0.4,0.2\rangle \\ \langle(13,23);0.6,0.2,0.5\rangle & \langle(13,13);0.8,0.4,0.1\rangle & \langle(13,-1);0.5,0.7,0.1\rangle & \langle(13,-11);0.5,0.3,0.5\rangle \\ \langle(23,23);0.7,0.3,0.3\rangle & \langle(23,13);0.4,0.6,0.3\rangle & \langle(23,-1);0.4,0.6,0.3\rangle & \langle(23,-11);0.7,0.4,0.2\rangle \end{bmatrix}$$

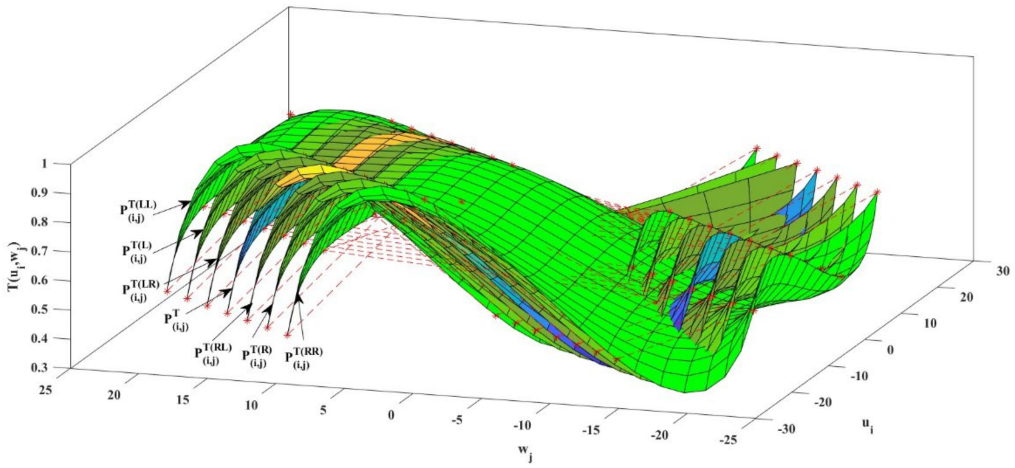

Figure 2. IT2NBSs for truth membership with its respective IT2NCN

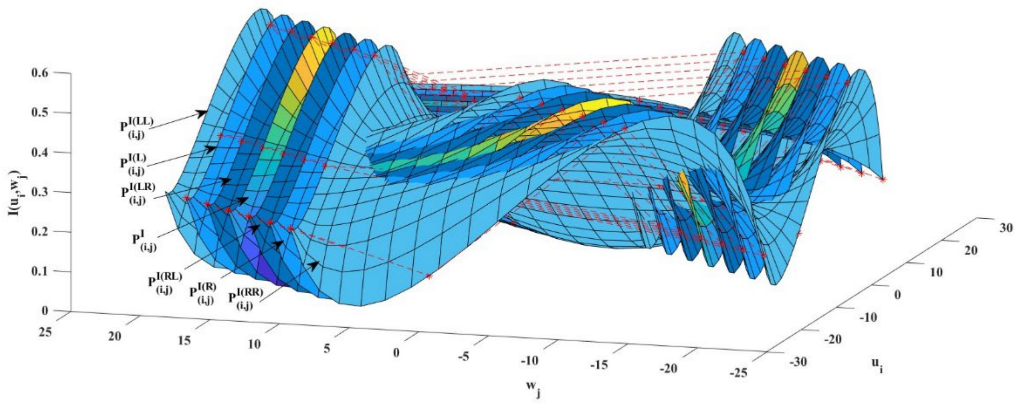

Figure 3. IT2NBSs for indeterminacy membership with its respective IT2NCN

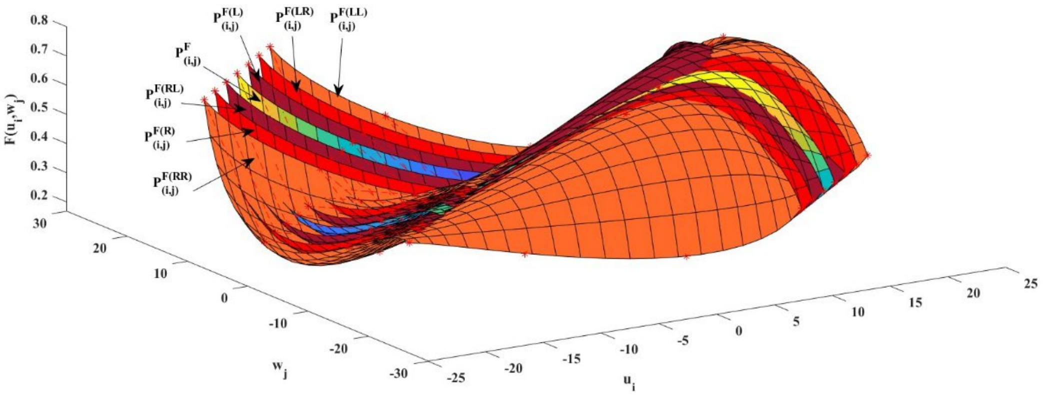

Figure 4. IT2NBSs for falsity membership with its respective IT2NCN

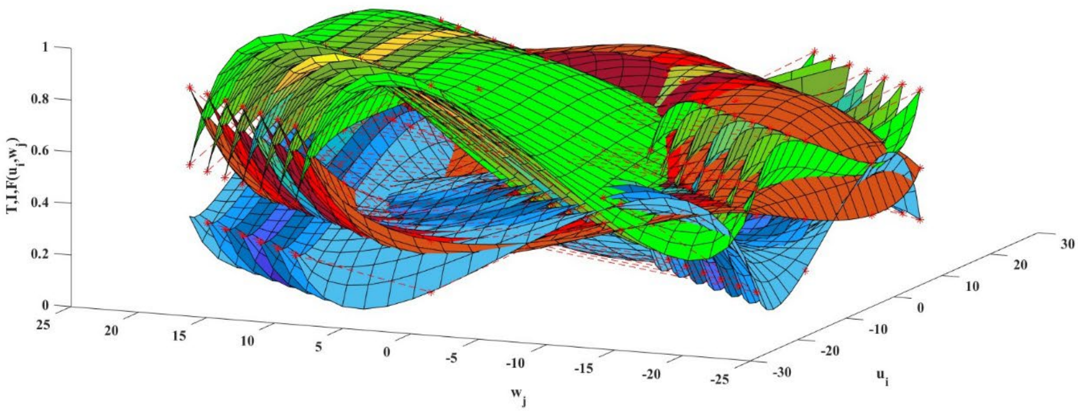

Figure 5. IT2NBSs with their respective IT2NCNs and memberships

Figure 2-4 depicts the IT2NBSs, which comprises truth, indeterminacy, and falsity membership for each of their left and right footprints. Truth membership is represented by the green curves, indeterminacy by the blue curves, and falsehood by the red curves. **Figure 5** displays the IT2NBSs with their respective IT2NCNs and memberships. **Figure 6** at the end of this section shows how an algorithm for creating the IT2NBSs works.

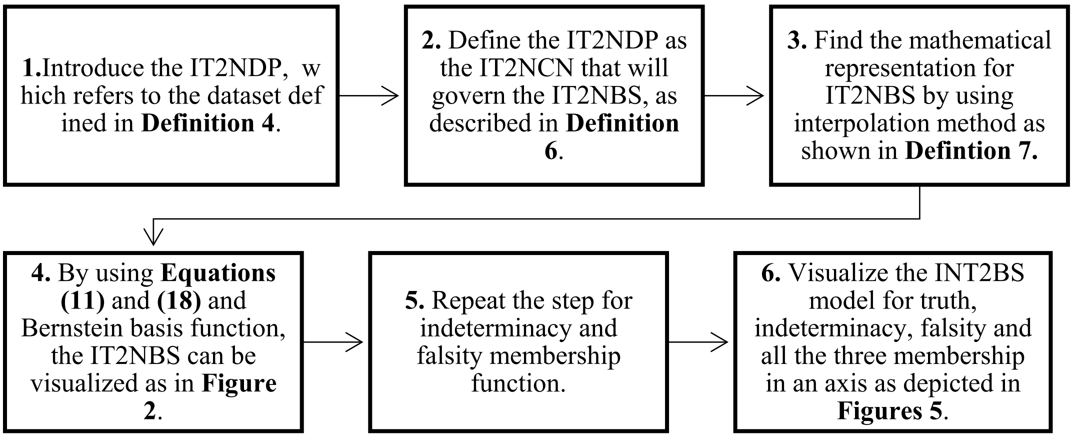

Figure 6. An algorithm to create the IT2NBS interpolation models.

5. Conclusion

This study proposed IT2NS principles for developing the IT2NCN, which governs IT2NBS behaviors. The model demonstrates how to display an uncertain dataset using IT2NS theory. This approach has the potential to make large contributions to fields with high levels of uncertainty, such as bathymetry data. Predictive models are utilized in a variety of medical applications, including cancer-level prediction, image-blurring detection, and catastrophe warning systems. This research can be expanded to incorporate more complex problems, particularly the type-2 neutrosophic set. Furthermore, different geometric models, such as B-spline and non-uniform rational B-spline (NURBS), could be used in future research to increase the study's visualization capabilities. Furthermore, this study can grow using surface modeling approaches or approximation methods.

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