Visualization of Picture Fuzzy Bézier Curve Interpolation Model with Its Properties

Nur Azlida Ahmad[1,2] and Mohammad Izat Emir Zulkifly[*2]

[1]Department of Foundation and General Studies, Universiti Tenaga National (UNITEN)
Jalan Ikram-Uniten, 43000 Kajang, Malaysia
[1,2]Department of Mathematics, Faculty of Sciences, University Teknologi Malaysia (UTM)
81310, Johor Bharu, Malaysia
[*]Corresponding author, E-mail: izatemir@utm.my, nurazlida@graduate.utm.my

Abstract

This paper proposed and visualized picture fuzzy Bézier curve model by using interpolation method. This model was presented using picture fuzzy sets theory as a basis and its properties. Based on the concept of picture fuzzy point, picture fuzzy number and picture fuzzy relation, the picture fuzzy point relation and picture fuzzy control point relation are defined. The picture fuzzy Bézier curve is created by blending the picture fuzzy control point relation with the Bernstein basis function. Finally, the picture fuzzy Bézier curve is used to interpolate and visualize the picture fuzzy data. Some numerical examples and the properties of the picture fuzzy Bézier curve model is also shown.

Keywords: *Picture fuzzy, Bézier curve, Interpolation, fuzzy Bézier curve, fuzzy point relation, fuzzy control point relation.*

1. Introduction

Curves and surfaces are necessary for modeling and designing complicated shapes and forms in a variety of industries, including automotive, aerospace, animation, and more, using Computer-aided geometric design (CAGD). Curves are used to define the shape and outline of objects. The most common types of curves used in CAGD are Bézier Curves, B-Spline and NURBS. Many authors have investigated different types of surfaces and curves, particularly Bezier curves in various fields such as in constructing a digital route model (Bogumil et al., 2024), design a Hamiltonian system and a gradient system (Cuesta and Alvarez., 2023), (Cui et al., 2023) developing an algorithmic approach for adaptive extension fitting and also in analyze the capability of Bézier curves (Rahmanović et al., 2023).

However, because of the uncertainty of the data, there are a number of issues that can occur when reconstructing curves (Zakaria et al., 2016; Fatah and Rozaimi, 2015; Zakaria and Wahab, 2014, 2012). Zadeh developed fuzzy set theory which is one method for identifying uncertainty data. (Azmi et al., 2023) involved in another method that applies the type-2 intuitionistic fuzzy set (T-2IFS) to complicated uncertainty data in geometric modeling, where the type-2 fuzzy concept is used to define the data. Fuzzy interpolation Bézier curves was used by (Zakaria et al., 2019) to verify the alphabet. Moreover, (Rosli and Zulkifly 2023; 2024) present the algorithm for the generating the interval of neutrosophic cubic Bézier curve (INCBC).

Nevertheless, there are difficulties in solving uncertainty data issues, such as the occurrence of refused data. (Cuong, et al., 2013, 2014) developed the picture fuzzy set, which is an extension of intuitionistic fuzzy set (IFS) (Li et al., 2005, 2008; Das et al., 2015, 2018) and fuzzy set (Zadeh, 1965). By establishing the membership degrees for refusal and neutrality, picture fuzzy set can easily handle the uncertain character of human thinking. In a picture fuzzy set, each element has degrees of membership, non-membership, and an additional degree representing neutrality and refusal, thus offering a more nuanced representation of uncertain information. By providing membership degree of neutral and refusing, it has immediate control over the fundamental uncertainty to the way the human brain develops. If membership degree neutral and refusal is zero, so PFS become as IFS. In situations like these, PFS is a preferable option because it offers membership degrees that are positive, neutral, negative, and refusal (Cuong et al., 2013, 2014). This is one scenario in which they might voice their opinions. In modelling, picture fuzzy Bézier curves are a recent innovation designed to handle uncertainty in data. This approach extends traditional fuzzy Bézier curves by incorporating picture fuzzy sets, which allow for an additional level of uncertainty management beyond typical fuzzy logic.

Consequently, visualization of picture fuzzy Bézier curve interpolation model with its properties involves defining control points with picture fuzzy values is presented in this research where it can manage uncertain data. The arrangement of this paper according to the introduction and earlier research on the picture fuzzy set method including geometric modeling were covered in Section 1. Section 2 contains preliminaries, such as definitions for picture fuzzy sets, picture fuzzy numbers, and picture fuzzy relations. Furthermore, in section 2, picture fuzzy point relation (PFPR) and picture fuzzy control point relation (PFCPR) are introduced. In Section 3, picture fuzzy Bézier curve (PFBC) interpolation using PFCPR introduced. Section 4 presents the numerical example of PFBC model, PFBC visualization and PFBC properties. Section 5 represents the research's conclusion and section 6 is the references.

2. Preliminaries

In order to study the topic of curve uncertainty, many definitions pertaining to fuzzy problems have been studied in order to construct a picture fuzzy Bézier curve interpolation model. The definition of fuzzy sets (Zadeh, 1965), intuitionistic fuzzy sets (Atanassov et al.,1983), and picture fuzzy sets (Cuong et al., 2013) are mentioned.

Definition 1: Assume that X is a nonempty set. A fuzzy set drawn from X is known as

$$A=\{(x,\mu_A(x)):x\in X\} \tag{1}$$

where $\mu_A(x):X\to[0,1]$ shows which members of the fuzzy set. A fuzzy set is a combination of items having varying degrees of membership.

Definition 2: An intuitionistic fuzzy set (IFS) A^* on a universe X is an object of the form

$$A^*=\{(x,\mu_{A^*}(x),v_{A^*}(x))\mid x\in X\} \tag{2}$$

where $\mu_{A^*}(x)\in[0,1]$ is referred to "membership degree of X in A^*, $v_{A^*}(x)\in[0,1]$ is called the "degree of non-membership of X in A^*" and μ_{A^*} and v_{A^*} satisfy the following condition :

$$0\le \mu_{A^*}(x)+v_{A^*}(x)\le 1 \text{ for all } x\in X \tag{3}$$

The hesitancy degree of element x in A^* is represented by $\pi_{A^*}(x)=1-[\mu_{A^*}(x)+v_{A^*}(x)]$. IFS(X) shall stand for the set of all IFS in X.

Definition 3: The form of a picture fuzzy set (PFS) set $\widehat{A}$ on a discourse universe X is

$$\widehat{A}=\{x,\mu_{\widehat{A}}(x),\eta_{\widehat{A}}(x),v_{\widehat{A}}(x)\mid x\in X\} \tag{4}$$

while $\mu_{\widehat{A}}(x)\in[0,1]$ is called the "degree of positive membership of x in $\widehat{A}$", $\eta_{\widehat{A}}(x)\in[0,1]$ is called the "degree of neutral membership of x in $\widehat{A}$", $v_{\widehat{A}}(x)\in[0,1]$ is called the "degree of negative membership of x in $\widehat{A}$", and where $\mu_{\widehat{A}}(x)$, $\eta_{\widehat{A}}(x)$ and $v_{\widehat{A}}(x)$ must satisfy the following condition:

$$0\le \mu_{\widehat{A}}(x)+\eta_{\widehat{A}}(x)+v_{\widehat{A}}(x)\le 1;\forall x\in X \tag{5}$$

In above definition, for all $x\in X$, $\rho_{\widehat{A}}(x)=1-[\mu_{\widehat{A}}(x)+\eta_{\widehat{A}}(x)+v_{\widehat{A}}(x)]$ could be addressed the "degree of refusal membership of X in $\widehat{A}$. If $\mu_{\widehat{A}}(x)=0$ for all $x\in X$, then the PFS reduces into IFS (Atanassov et al., 1983)

Definition 4: The picture fuzzy number(PFN) for a fixed $x\in\widehat{A}$ is $(\mu_{\widehat{A}}(x),\eta_{\widehat{A}}(x),v_{\widehat{A}}(x),\rho_{\widehat{A}}(x))$, where $\mu_{\widehat{A}}(x),\eta_{\widehat{A}}(x),v_{\widehat{A}}(x),\rho_{\widehat{A}}(x)\in[0,1]$ and $\mu_{\widehat{A}}(x)+\eta_{\widehat{A}}(x)+v_{\widehat{A}}(x)+\rho_{\widehat{A}}(x)=1$. Simply, PFN is represented as $(\mu_{\widehat{A}}(x),\eta_{\widehat{A}}(x),v_{\widehat{A}}(x))$.

Definition 5: Assume that typical non-empty sets are X, Y and Z. A relation of picture fuzzy (PFR), $\widehat{R}$ is a subset of picture fuzzy, $X\times Y$ given by

$$\widehat{R}=\{((x,y),\mu_{\widehat{R}}(x,y),\eta_{\widehat{R}}(x,y),v_{\widehat{R}}(x,y)):x\in X,y\in Y\} \tag{6}$$

whereas $\mu_{\widehat{R}}:X\times Y\to[0,1]$, $\eta_{\widehat{R}}:X\times Y\to[0,1]$ and $v_{\widehat{R}}:X\times Y\to[0,1]$ fulfilled the requirement

$$0\le \mu_{\widehat{R}}(x,y)+\eta_{\widehat{R}}(x,y)+v_{\widehat{R}}(x,y)\le 1 \text{ for } (x,y)\in(X,Y) \tag{7}$$

$PFR(X\times Y)$ represents the collection of all picture fuzzy relations in $X\times Y$.

2.1 Picture Fuzzy Point Relation

A relation of fuzzy point refers to a concept in fuzzy set theory and fuzzy logic where relationships between points are defined with a degree of uncertainty or fuzziness. (Zhang and Xu, 2012) defined several new intuitionistic preference relations, such as the consistent and incomplete intuitionistic preference relations and their properties were examined. As a result, (Cuong et al., 2013, 2014) presents some initial picture fuzzy relation (PFR) results. Definition 6 describes the relation of picture fuzzy point (PFPR) according to the PFS.

Definition 6: If P and Q are sets of non-empty points and $P,Q,I \subseteq \mathbb{R}^3$ then PFPR is defined as

$$\widehat{S}=\left\{\begin{array}{l}\left\langle\left(p_i,q_j\right),\mu_{\widehat{S}}\left(p_i,q_j\right),\eta_{\widehat{S}}\left(p_i,q_j\right),v_{\widehat{S}}\left(p_i,q_j\right),\rho_{\widehat{S}}\left(p_i,q_j\right)\right\rangle \mid \\ \left(p_i,q_j\right),\mu_{\widehat{S}}\left(p_i,q_j\right),\eta_{\widehat{S}}\left(p_i,q_j\right),v_{\widehat{S}}\left(p_i,q_j\right),\rho_{\widehat{S}}\left(p_i,q_j\right)\in I\end{array}\right\} \tag{8}$$

In this case $\left(p_i,q_j\right)$ represent the point in an ordered pair and $\left(p_i,q_j\right)\in P\times Q$. Whereas the matching ordered pair of points in $[0,1]\in I$, that represent positive, neutral, negative and refusal membership are $\mu_{\widehat{S}}\left(p_i,q_j\right)$, $\eta_{\widehat{S}}\left(p_i,q_j\right)$, $v_{\widehat{S}}\left(p_i,q_j\right)$ and $\rho_{\widehat{S}}\left(p_i,q_j\right)$. The level of refusal is demonstrated by

$$\rho_{\widehat{S}}(x)=1-\left[\mu_{\widehat{S}}\left(p_i,q_j\right)+\eta_{\widehat{S}}\left(p_i,q_j\right)+v_{\widehat{S}}\left(p_i,q_j\right)\right] \tag{9}$$

where $\rho_{\widehat{S}}(x)$ indicates the level of refusal and the rule $0\le\mu_{\widehat{S}}\left(p_i,q_j\right)+\eta_{\widehat{S}}\left(p_i,q_j\right)+v_{\widehat{S}}\left(p_i,q_j\right)\le 1$ has been observed to.

2.2 Picture Fuzzy Control Point Relation

In curve modeling, a fuzzy control point refers to a control point whose position or influence on the curve is characterized by fuzzy sets, rather than being precisely defined. This concept is particularly useful in scenarios where data is uncertain or imprecise, allowing for more flexible and robust curve representations. The notion of fuzzy control point from (Wahab and Zulkifly, 2018a, 2018b) is used in this part to develop the picture fuzzy control point relation (PFCPR) in definition 7.

Definition 7: Let $\widehat{S}$ be a PFPR, which is used to characteristic the curve and is represented by coordinates and position as $\widehat{C}_i$. It is thought of as a group of points $n+1$.

$$\widehat{C}_i=\left\{\widehat{C}_0,\widehat{C}_1,\widehat{C}_2,..,\widehat{C}_n\right\} \tag{10}$$

3. Picture Fuzzy Bézier Curve Interpolation

In this section, the picture fuzzy Bézier curves interpolation which are based on fuzzy Bézier curves is discussed. As seen in the following definition, PFBC is produced by combining PFCP with the basis function or Bernstein polynomial. The properties of picture fuzzy Bézier curve also discussed in this section.

3.1 Picture Fuzzy Bézier Curve

Definition 8: Let $\widehat{C}_i=\left\{\widehat{C}_i^n\right\}$ for $i=0,1,2,..,n$ represent the PFCP as well PFBC be represented by $\widehat{B}_i(t)$ with a vector of position across the curve as a function of the scalar t, therefore after combining PFBC with $\widehat{C}$ using the blending function, it is expressed as

$$\widehat{B}_i(t)=\sum_{i=0}^{n}\widehat{C}_i\beta_i^n(t),\ 0\le t\le 1 \tag{11}$$

where $\beta_i^n(t)=\binom{n}{i}t^i(1-t)^{n-i}\equiv 1$ are the Bernstein polynomial with degree n and the blending function with $\binom{n}{i}=\frac{n!}{i!(n-i)!},0\equiv 1$ is the binomial coefficient. The geometric coefficient, $\widehat{C}_i$ that defines the control polygon for the PFBC degree n is the i^{th} PFCP. The notation for PFBC with the n^{th} degree is

$$\widehat{B}(t)=\widehat{C}_0\beta_0^n+\widehat{C}_1\beta_1^n+\widehat{C}_2\beta_2^n+..+\widehat{C}_n\beta_i^n \quad (12)$$

The PFBC in (11) is represented as follows and consists of positive membership, neutral membership, negative membership, and refusal curve:

$$\widehat{B}^{\mu}(t)=\sum_{i=0}^{n}\widehat{C}_i\beta_i^n(t) \quad (13)$$

$$\widehat{B}^{\eta}(t)=\sum_{i=0}^{n}\widehat{C}_i\beta_i^n(t) \quad (14)$$

$$\widehat{B}^{\nu}(t)=\sum_{i=0}^{n}\widehat{C}_i\beta_i^n(t) \quad (15)$$

$$\widehat{B}^{\rho}(t)=\sum_{i=0}^{n}\widehat{C}_i\beta_i^n(t) \quad (16)$$

A polygon known as the PFCP for PFBC is created by joining the PFCP in the proper sequence. PFBC typically takes the form of PFCP. The initial and final points of PFCP align with the initial and final PFCP.

3.2 Picture Fuzzy Bézier Curve Interpolation

The technique of approximating values that are unknown but fall inside the range of known data points is called interpolation. Consequently, The objective of curve interpolation is to create a smooth curve that goes through a specified set of points (D Solomon, 2006). Each point on the PFBC has a corresponding parameter. Assume that $\widehat{D}_i$ be a fuzzy data connected to the scalar $t_i=i/n$. So that, PFBC interpolation might be stated as

$$\widehat{B}(t_i)=\widehat{D}_i \quad (17)$$

Consequently, by using equation (17),

$$\begin{gathered}\widehat{D}_0=\beta_0^n(t_0)\widehat{C}_0+\beta_1^n(t_0)\widehat{C}_1+\beta_2^n(t_0)\widehat{C}_2+..+\beta_n^n(t_0)\widehat{C}_n\\ \widehat{D}_1=\beta_0^n(t_1)\widehat{C}_0+\beta_1^n(t_1)\widehat{C}_1+\beta_2^n(t_1)\widehat{C}_2+..+\beta_n^n(t_1)\widehat{C}_n\\ \vdots\\ \widehat{D}_n=\beta_0^n(t_n)\widehat{C}_0+\beta_1^n(t_n)\widehat{C}_1+\beta_2^n(t_n)\widehat{C}_2+..+\beta_n^n(t_n)\widehat{C}_n\end{gathered} \quad (18)$$

Since Bézier curve interpolate the end points, $\widehat{C}_0$ and $\widehat{C}_n$ in every case, so that $\widehat{C}_0=\widehat{D}_0$ and $\widehat{C}_n=\widehat{D}_n$.
Matrix form can be obtained using the system Equation (18) as

$$\left[\widehat{D}\right]=[\beta]\left[\widehat{C}\right] \quad (19)$$

where

$$\begin{gathered}\left[\widehat{D}\right]^T=\left[\widehat{D}_0(t_0)\quad \widehat{D}_1(t_1)\quad \ldots\quad \widehat{D}_n(t_n)\right]\\ [\beta]=\begin{bmatrix}\beta_0^n(t_0) & \cdots & \cdots & \beta_n^n(t_0)\\ \vdots & \ddots & & \vdots\\ \vdots & & \ddots & \vdots\\ \beta_0^n(t_n) & \cdots & \cdots & \beta_n^n(t_n)\end{bmatrix}\\ \left[\widehat{C}\right]^T=\left[\widehat{C}_0\quad \widehat{C}_1\quad \ldots\quad \widehat{C}_n\right]\end{gathered} \quad (20)$$

then, the control polygon is then immediately constructed using an inverse matrix such as β which is squared matrix.

$$\left[\widehat{C}_i\right]=[\beta]^{-1}\left[\widehat{D}_i\right] \tag{21}$$

Thus, using equation (21) can achieve PFBC interpolation.

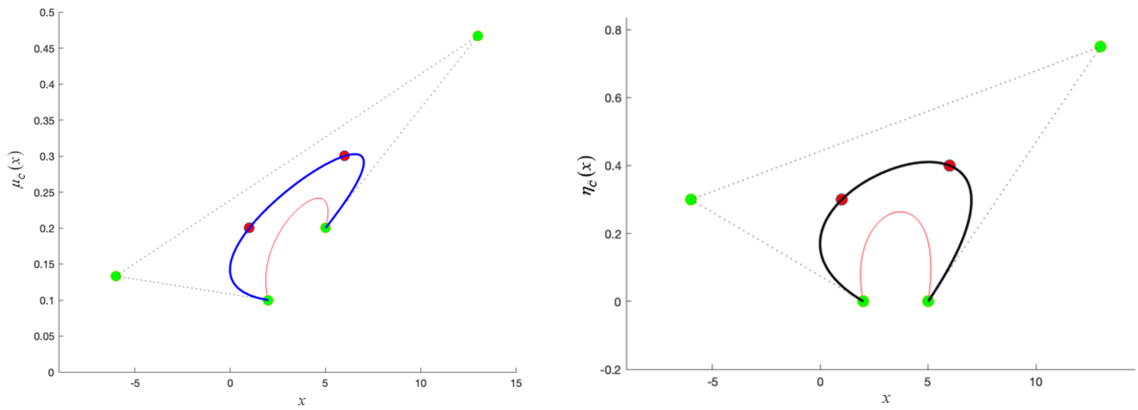
4. Visualization of Picture Fuzzy Bézier Curve Interpolation

In this section, the picture fuzzy Bézier curve interpolation model for negative, neutral, positive and refusal membership will be visualized. Table 1 shows the PFCPR for each membership. All values of PFCPR simply using x value.

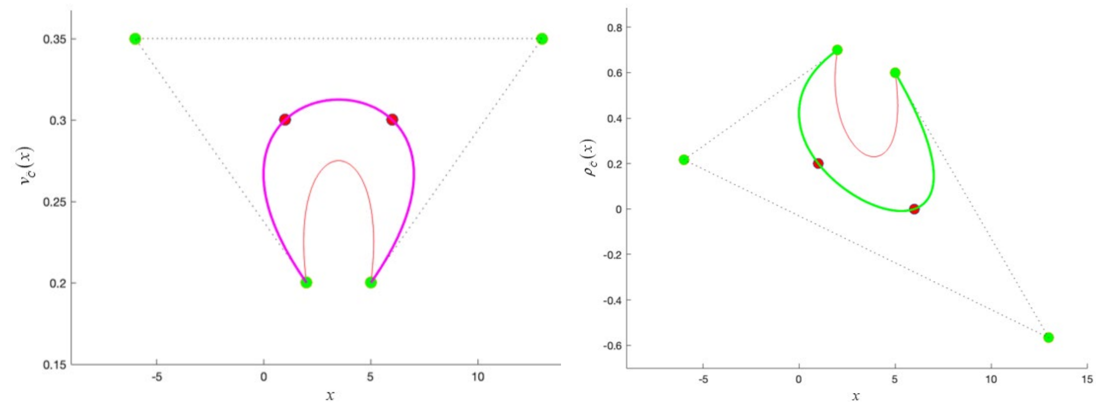
Table 1 PFCPR and its corresponding memberships.

PFCPR, $\widehat{C}_i$	Positive membership, $\mu_{\widehat{C}}(x)$	Neutral membership, $\eta_{\widehat{C}}(x)$	Negative membership, $\nu_{\widehat{C}}(x)$	Refusal membership, $\rho_{\widehat{C}}(x)$
2	0.1	0.0	0.2	0.7
1	0.2	0.3	0.3	0.2
6	0.3	0.4	0.3	0.0
5	0.2	0.0	0.2	0.6

Figure 1(a), (b), (c) and (d) display the picture fuzzy Bézier curve interpolation for positive, neutral, negative and refusal membership respectively. The red dotes denotes as the picture control point relation, the green dots denote as interpolation of PFCP, the black dash line denotes as control polygon for interpolation of PFCP, the black line is Picture fuzzy Bézier curve interpolation and the red line denotes as picture fuzzy Bézier curve approximation. Figure 2 visualizes the picture PFBC interpolation for all memberships on one axis and different views of PFBC.



(a) (b)



(c) (d)

Figure 1 PFBC interpolation along with the corresponding control polygon and control point for all memberships

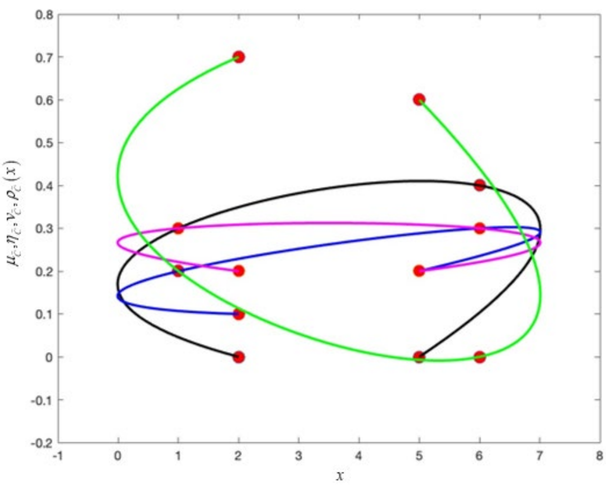


Figure 2 PFBC interpolation for all membership

Based on the results of figure 1, the shape of the curve for each memberships changes according to the uncertainty value that has been set. This result additionally illustrates the features of PFS theory. The three novelties of this research are firstly, the PFPR for PFBC was introduced, secondly the properties for PFBC were analyzed and determined and last the visualization of PFBC interpolation were presented.

Bézier curve generation is determined by its control polygon. Certain characteristics of Bézier curves are clearly identifiable since the Bézier basis and the Bernstein basis are the same. Therefore, based on the concepts of fundamental Bézier basis features the PFBC has the following essential properties.

1) One smaller than the total number of points in the picture fuzzy control polygon is the degree of the polynomial that defines the segment of the picture fuzzy curve.
2) In general, the picture fuzzy control polygon's shape is followed by the PFBC.
3) The initial and last points of the picture fuzzy control polygon and the first and last points of the picture fuzzy curve coincide.
4) The direction of the tangent vectors at the ends of the fuzzy curve in the image is the same as that of the first and last polygons, respectively.

5. Conclusions

In this study, through PFCPR, picture fuzzy Bézier curves interpolation was introduced. Considering that it has a positive, neutral, negative and refusal membership, the PFBC model interpolation is a suitable method in handling uncertainties. This research's significant contribution is that it has shown that these functions can be used to examine and process any type of data. Other from that, this model's advantages include better handling of uncertain data, the ability to produce flexible and smooth Bézier curves that appropriately express complex shapes, and the ability to smoothly interpolate uncertain data points. As a result, this model may be expanded to use B-spline and NURBS modeling to address the picture data problem. Overall, the PFBC interpolation model greatly improves the capacity to handle and model uncertain data in a variety of industries (forecasting, robotics, CAD, and CAGD), which promotes improved application dependability and optimization as well as better decision-making.

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