Mathematical Demonstration on Neutron Thermalization by a Rich H-atom Polymer Materials, PETE fiber

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ABSTRACT

Neutrons would be thermalized by some rich H-atom polymer materials such as polyethyleneterephthalate, PETE fibers because of mathematical demonstration have proved it clearly. Vector diagram analysis, law of momentum conservation and law of mechanical energy conservation were also used in this mathematical physics proving. According to the result, writer found that, all energy from neutrons would be transferred out of a neutrons into proton particles that confirming by laboratory testing.

Keywords: Neutron Thermalization, Polymer Materials, Mathematical Demonstration

1. Introduction

In the future, researcher will use polyethyleneterephthalate, PETE fibers for neutron attenuation in order to make shielding concrete materials that will be applied in the nuclear power plant or in the hospital. Because of neutron particles are normally dangerous to human body such as they will affect blood system or tissue system as be cancer (Cember, 1992).This polymer material (PETE) was selected because it has high ratio of H-atom per molecule (Odian, 2004). Researcher expected that this polymer will be used for neutron attenuation effectively. In the similarity procedures of (Lamarsh, 1975) and (Jewett/Serway, 2008), Vector diagram analysis, law of momentum conservation and law of mechanical energy conservation were also applied in this mathematical physics expressions as physiochemical hypothesis. Finally, real shielding testing was be tested by laboratory for comparing in appropriated statistical confidence.

2. Objectives of the study

This research focused on finding the new mathematical method for investigation the ability of neutron attenuation of PETE fiber and comparing with the real condition of neutron shielding in laboratory.

3. Materials and methods

There are two section in this studying, (1) mathematical proving and (2) laboratory testing

□ **Mathematical Analytical Procedures**

A simply situation can be considered below; first, a neutron mass m has an initial energy and linear momentum E_1 and $\vec{P}_1$ respectively. And then, it moves to collide with a rest proton mass M which has no an initial energy and linear momentum. After collision, the neutron scattered from the central line with an angle θ_1 which leads to E_2 and $\vec{P}_2$ remaining, whereas the proton mass M remains E_A and $\vec{P}_A$ which has θ_2 as a scattered angle.

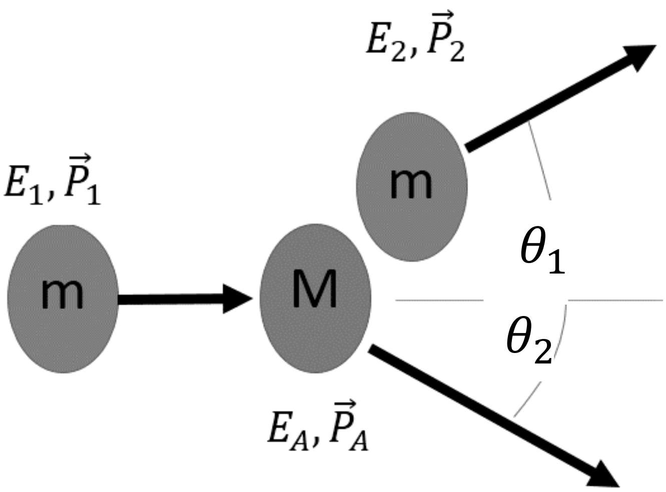


Figure 1: a proton is collided by a neutron

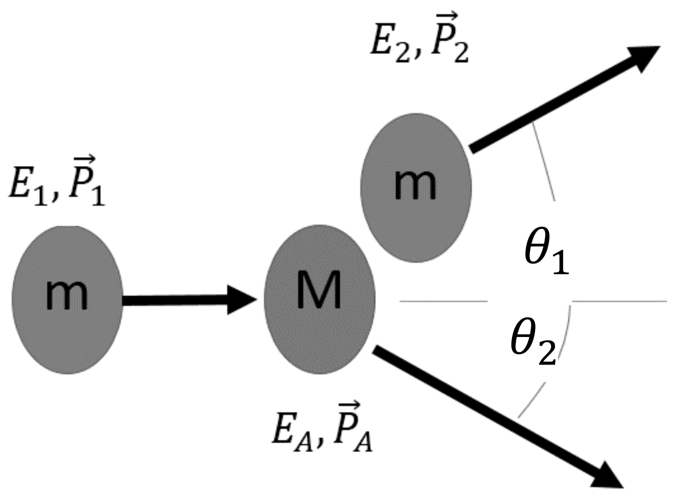
According to the law of energy conservation; $E_1 = E_2 + E_A$ and According to the law of linear momentum conservation; $\vec{P}_1 = \vec{P}_2 + \vec{P}_A$ Then, we can rewrite figure 1 to be a vector illustration and then, solving to find the final energy of neutron

Laboratory Testing

BF_3 proportional tube detector was used for neutron attenuation ability of materials (fibered composite concrete). Fiber reinforced concretes were prepared by mixing between concretes and fibers in size 10x10x10 cm length. Two sizes of fiber were used in this mixing (1.3D and 25D). The fiber content in concrete was divided into four system; OPC (concrete without fiber), FRC01 (concrete with 0.1% fiber), FRC02 (concrete with 0.2% fiber), and FRC03 (concrete with 0.3% fiber).

4. Results

A simply situation can be considered below; first, a neutron mass m has an initial energy and linear momentum E_1 and $\vec{P}_1$ respectively. And then, it moves to collide with a rest proton mass M which has no an initial energy and linear momentum. After collision, the neutron scattered from the central line with an angle θ_1 which leads to E_2 and $\vec{P}_2$ remaining, whereas the proton mass M remains E_A and $\vec{P}_A$ which has θ_2 as a scattered angle.



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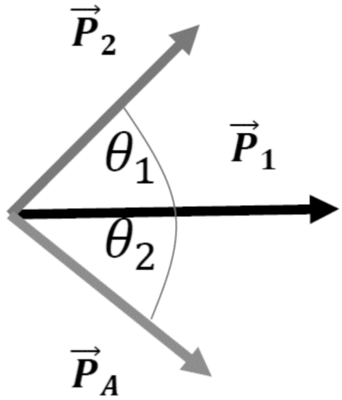


Figure 2: momentum expression as vector diagram

Forasmuch, the cosine's law can be applied to from the new mathematical relation;

$$P_A^2 = P_1^2 + P_2^2 - 2P_1P_2\cos\theta_1 \qquad \text{------- (1)}$$

According to the classical mechanics, the quantity of momentum can be written as

$$P = mv$$

Thus

$$P_A = Mv_A$$

$$P_A^2 = (Mv_A)^2$$

$$P_A^2 = M^2 v_A^2 \qquad \text{------- (2)}$$

Since

$$E_A = \frac{1}{2}Mv_A^2$$

So

$$v_A^2 = \frac{2E_A}{M} \qquad \text{------- (3)}$$

Substitute (3) into (2)

$$P_A^2 = M^2 \left(\frac{2E_A}{M}\right)$$

$$P_A^2 = 2ME_A \qquad \text{------- (4)}$$

In the same way, using the previous procedure; now it can be derived as

$$P_1^2 = 2mE_1 \qquad \text{------- (5)}$$

And

$$P_2^2 = 2mE_2 \qquad \text{------- (6)}$$

Substitute (4), (5) and (6) into (1);

$$2ME_A = 2mE_1 + 2mE_2 - 2\sqrt{2mE_1}\sqrt{2mE_2}\cos\theta_1$$

$$2ME_A = 2mE_1 + 2mE_2 - 2(2)m\sqrt{E_1E_2}\cos\theta_1$$

Multiply by $\frac{1}{2}$ both sides

So, $$ME_A = mE_1 + mE_2 - 2m\sqrt{E_1E_2}\cos\theta_1 \qquad \text{------- (7)}$$

It decisively that $\frac{M}{m} \equiv A$

Where M is the target atomic mass

m is the neutron mass (around 1 a.m.u.)

And A is an atomic mass number

So, $$M = A \qquad \text{------ (8)}$$

Substitute (8) into (7);

$$AE_A = mE_1 + mE_2 - 2m\sqrt{E_1E_2}\cos\theta_1$$

$$AE_A = (1)E_1 + (1)E_2 - 2(1)\sqrt{E_1E_2}\cos\theta_1$$

$$AE_A = E_1 + E_2 - 2\sqrt{E_1E_2}\cos\theta_1 \qquad \text{------ (9)}$$

Then, substitute $E_A = E_1 - E_2$ into (9)

$$A(E_1 - E_2) = E_1 + E_2 - 2\sqrt{E_1E_2}\cos\theta_1$$

$$AE_1 - AE_2 = E_1 + E_2 - 2\sqrt{E_1E_2}\cos\theta_1$$

$$-AE_2 - E_2 + 2\sqrt{E_1E_2}\cos\theta_1 + AE_1 - E_1 = 0$$

$$AE_2 + E_2 - 2\sqrt{E_1E_2}\cos\theta_1 - AE_1 + E_1 = 0$$

$$(A+1)E_2 - 2\sqrt{E_1E_2}\cos\theta_1 - (A-1)E_1 = 0$$

$$(A+1)E_2 - 2\sqrt{E_1}\sqrt{E_2}\cos\theta_1 - (A-1)E_1 = 0$$

$$(A+1)\sqrt{E_2}^{\,2} - 2\sqrt{E_1}\sqrt{E_2}\cos\theta_1 - (A-1)E_1 = 0 \qquad \text{----- (10)}$$

Then, the following quadratic equation below can be used to find the roots of the former equation (10);

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$(A+1)\left(\sqrt{E_2}\right)^2 - 2\sqrt{E_1}\sqrt{E_2}\cos\theta_1 - (A-1)E_1 = 0$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1}\cos\theta_1 \pm \sqrt{\left(2\sqrt{E_1}\cos\theta_1\right)^2 - 4(A+1)(-(A-1))E_1}}{2(A+1)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1}\cos\theta_1 \pm \sqrt{4E_1\cos^2\theta_1 + 4(A+1)((A-1))E_1}}{2(A+1)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1}\cos\theta_1 \pm \sqrt{4E_1\cos^2\theta_1 + 4(1+1)((1-1))E_1}}{2(1+1)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1}\cos\theta_1 \pm \sqrt{4E_1\cos^2\theta_1 + 4(2)(0)E_1}}{2(2)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1}\cos\theta_1 \pm \sqrt{4E_1\cos^2\theta_1 + 0}}{2(2)}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1}\cos\theta_1 \pm \sqrt{4E_1\cos^2\theta_1}}{4}$$

$$\sqrt{E_2} = \frac{2\sqrt{E_1}\cos\theta_1 \pm 2\sqrt{E_1}\cos\theta_1}{4}$$

$$\sqrt{E_2} = \frac{\sqrt{E_1}\cos\theta_1 \pm \sqrt{E_1}\cos\theta_1}{2}$$

$$\sqrt{E_2} = \frac{1}{2}\left[\sqrt{E_1}\cos\theta_1 \pm \sqrt{E_1}\cos\theta_1\right]$$

$$E_2 = \frac{1}{4}\left[\sqrt{E_1}\cos\theta_1 \pm \sqrt{E_1}\cos\theta_1\right]^2$$

$$E_2 = \frac{1}{4}\left[\sqrt{E_1}(\cos\theta_1 \pm \cos\theta_1)\right]^2$$

Since the term $\cos\theta_1 + \cos\theta_1$ cannot be written due to physics' principle, thus use

$$E_2 = \frac{1}{4}\left[\sqrt{E_1}\,(\cos\theta_1 - \cos\theta_1)\right]^2$$

$$E_2 = \frac{1}{4}\left[\sqrt{E_1}\,(0)\right]^2$$

$$E_2 = 0 \qquad \text{--------- (11)}$$

According to the molecular formula of PETE; $(C_{10}H_8O_4)_n$, the 10st equation;

$(A+1)\sqrt{E_2}^{\,2} - 2\sqrt{E_1}\sqrt{E_2}\cos\theta_1 - (A-1)E_1 = 0$ was used to calculate the value of neutron attenuation ability of carbon and oxygen atoms as shows below by using Suphat's model (Suphat, 2007)

$$(A+1)\sqrt{E_2}^{\,2} - 2\sqrt{E_1}\sqrt{E_2}\cos\theta_1 - (A-1)E_1 = 0$$

According to Suphat's model, previously equation could be simplified as

$$E_2 = \frac{E_1}{(A+1)^2}\ [\cos\theta_1 + (A^2 \ \ [1 \ \ \cos^2\theta_1])^{\frac{1}{2}}]^2$$

For carbon atom; $E_2 = \frac{E_1}{(A+1)^2}\ [\cos\theta_1 + (A^2 - [1 - \cos^2\theta_1])^{\frac{1}{2}}]^2$

$$E_2 = \frac{E_1}{(12+1)^2}\ [\cos\theta_1 + (12^2 - [1 - \cos^2\theta_1])^{\frac{1}{2}}]^2$$

$$E_2 = \frac{E_1}{(13)^2}\ [\cos 90 + (144 - [1 - \cos^2 90])^{\frac{1}{2}}]^2$$

$$E_2 = \frac{E_1}{169}\ [0 + (144 - [1 - 0])^{\frac{1}{2}}]^2$$

$$E_2 = 0.846E_1 \qquad \text{(12)}$$

For oxygen atom; $E_2 = \frac{E_1}{(A+1)^2}\ [\cos\theta_1 + (A^2 - [1 - \cos^2\theta_1])^{\frac{1}{2}}]^2$

$$E_2 = \frac{E_1}{(16+1)^2}\ [\cos\theta_1 + (16^2 - [1 - \cos^2\theta_1])^{\frac{1}{2}}]^2$$

$$E_2 = \frac{E_1}{289}\ [\cos 90 + (256 - [1 - \cos^2 90])^{\frac{1}{2}}]^2$$

$$E_2 = \frac{E_1}{289}\ [0 + (256 - [1 - 0])^{\frac{1}{2}}]^2$$

$$E_2 = 0.882E_1 \qquad \text{(13)}$$

According to equation (11)-(13), the average value of neutron attenuation could be calculated by using weight average

$$\text{So, } \bar{E}_2 = \frac{0E_1 + (0.846E_1)(10) + (0.882E_1)(4)}{22}$$

$$\bar{E}_2 = 0.545E_1 \qquad (14)$$

□ Laboratory Testing

BF_3 proportional tube detector was used for neutron attenuation ability of materials (fibered composite concrete)

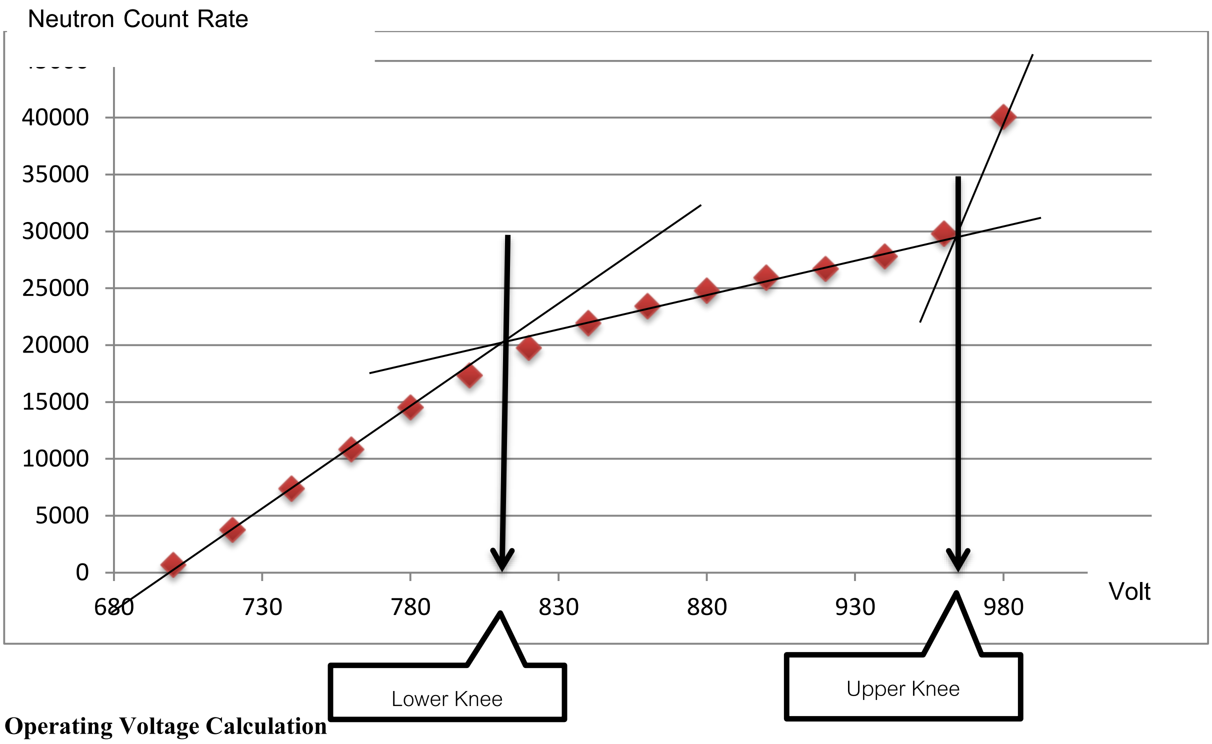
Fiber reinforced concretes were prepared by mixing between concretes and fibers in size 10x10x10 cm length. Two sizes of fiber were used in this mixing (1.3D and 25D). The fiber content in concrete was divided into four system; OPC (concrete without fiber), FRC01 (concrete with 0.1% fiber), FRC02 (concrete with 0.2% fiber), and FRC03 (concrete with 0.3% fiber).

Operating Voltage Experiment & Setting Up of Neutron Attenuation Ability Messurement

Starting Voltage = 700 volts

High Voltage (V)	counts / minute
700 (starting voltage)	700
720	3749
740	7401
760	10840
780	14542
800	17358
820	19770
840	21945
860	23436
880	24796
900	25933
920	26721
940	27810
960	29828
980	40092

The graph shows relation between voltage and neutron count rate



Operating Voltage Calculation

Optimum Voltage = ¼ (plateau length)

= ¼ (Upper Knee-Lower Knee)

= ¼ (920-818) = 844 volt

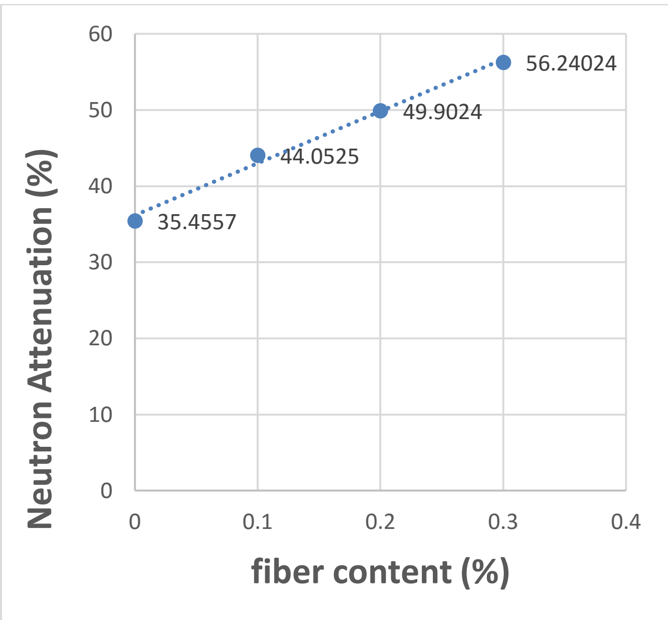
The result of neutron attenuation of concretes (fiber size = 1.3 D)

- Background Neutron = 22 count/10 minutes
- Non-shielding = 3416 counts/ 10 minutes

EXPERIMENTS	OPC	FRC01	FRC02	FRC03
1	2151	1924	1723	1482
2	2247	1966	1766	1501
3	2108	1875	1700	1498
4	2442	1935	1717	1480
5	2095	1893	1663	1483
6	2186	1874	1699	1525
Average	2204.833	1911.167	1711.333	1494.833
Different	1211.167	1504.833	1704.667	1921.167
Ratio	0.354557	0.440525	0.499024	0.562402
% Attenuation	35.4557	44.0525	49.90242	56.24024

Annotation; OPC = concrete without fibers, FRC01 = concrete with 0.1% fiber, FRC02 = concrete with 0.2% fiber, and FRC03 = concrete with 0.3% fiber

The graph below shows the relation between neutron attenuation of concrete and fiber content



Statistical Analysis

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

$H_1: \mu_i \neq \mu_j$ at least one pair, when $i \neq j$

Statistical Significant = 0.05

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Column 1	6	13229	2204.833	16543.77
Column 2	6	11467	1911.167	1351.767
Column 3	6	10268	1711.333	1154.667
Column 4	6	8969	1494.833	296.5667

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	1641032	3	547010.7	113.096	1.03E-12	3.098391
Within Groups	96733.83	20	4836.692			
Total	1737766	23				

Summary of Statistical Testing of 1.3 D fiber

Since F > F critical (H_0 is rejected), so; the values of neutron attenuation of concrete depends on the fiber content with 95% statistical confidence

The result of neutron attenuation of concretes (fiber size = 25 D)

- Background Neutron = 18 count/10 minutes
- Non-shielding = 3412 counts/ 10 minutes

EXPERIMENTS	FRC01	FRC02	FRC03
1	1916	1750	1432
2	1955	1760	1447
3	1864	1704	1402
4	1899	1715	1436
5	1901	1644	1480
6	1921	1636	1426
Average	1909	1702	1437
Different	1503	1711	1975
Ratio	0.440	0.501	0.579
% Attenuation	44.0	50.1	57.9

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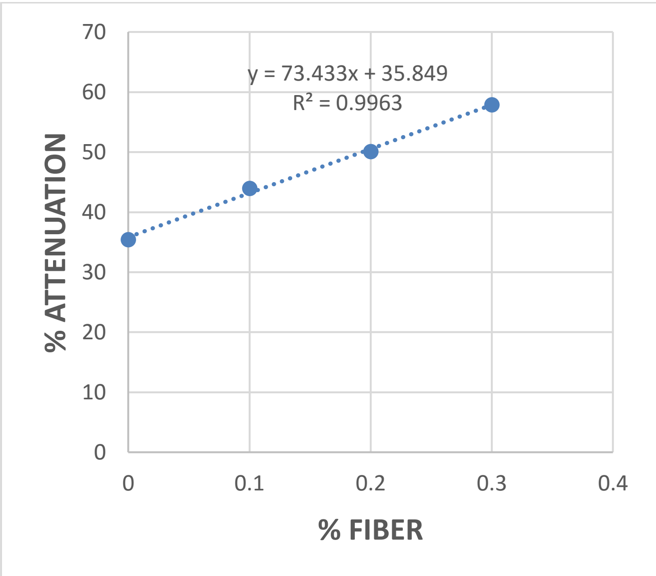

Statistical Analysis

$H_0: \mu_1 = \mu_2 = \mu_3$

$H_1: \mu_i \neq \mu_j$ at least one pair, when $i \neq j$

Statistical Significant = 0.05

Anova: Single Factor

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Column 1	6	11456	1909.333	899.4667
Column 2	6	10209	1701.5	2711.9
Column 3	6	8623	1437.167	664.1667

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*
Between Groups	672016.3	2	336008.2	235.7658	4.65E-12	3.68232
Within Groups	21377.67	15	1425.178			
Total	693394	17				

Summary of Statistical Testing of 25 D fiber

Since F > F critical (H_0 is rejected), so; the values of neutron attenuation of concrete depends on the fiber content with 95% statistical confidence

5. Discussion and Conclusion

According to the previous mathematical demonstration hypothesis, it shows that all energy from neutron will be transferred out of a neutron into proton particle as can be seen from equation (11). Because of polyethylene family is one of the rich H-atom polymers, so they probably can thermalize a neutron satisfactory which agree with (Cember, 1992) that used polyethylene for neutron shielding. According to calculation in the previously steps, it shows that PETE fiber can thermalized energy of neutron around 45.5% ($\overline{E}_2 = 0.545E_1$). The result form laboratory testing can be confirmed that the calculation result is realiable.

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